



ST. BRIGID'S
COLLEGE

Mathematics Specialist Units 3 & 4
Test 3 2017

Section 1 Calculator Free

Vectors in Two & Three Dimensions and Systems of Equations

STUDENT'S NAME: _____

DATE Term 2 Week 1

TIME: 20 minutes

MARKS: 19

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape,
ruler, highlighters, Formula Sheet.
Questions or parts of questions worth more than 2 marks require working to be shown to
receive full marks.

1. (4+2=6 marks)

(a) Solve the following system of equations.

$$2x - y + 2z = 1$$

$$x + y - 2z = 2$$

$$x - 2y + 4z = -1$$

(b) Hence explain what the equations and their solutions represent in space.

2. (6 marks)

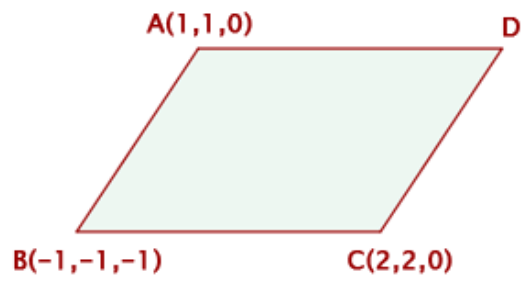
Determine the coordinates of the point of intersection of the line

$$L: \quad x + 1 = \frac{y - 2}{4} = z - 3 \quad \text{with the plane}$$

$$\Pi: \quad \tilde{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 8$$

3. (3+4=7 marks)

The diagram below shows a parallelogram ABCD .



Determine

(a) the coordinates of D

(b) the area of the parallelogram ABCD.

End of Section 1



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Mathematics Specialist Units 3 & 4
Test 3 2017

Section 2 Calculator Assumed

Vectors in Two & Three Dimensions and Systems of Equations

STUDENT'S NAME: _____

DATE: Term 2 Week 1

TIME: 35 minutes

MARKS: 31

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters,
Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on two sides of a single A4 page
Questions or parts of questions worth more than 2 marks require working to be shown to
receive full marks.

4. (4+2=6 marks)

(a) Determine the vector equation of each sphere $\Pi 1$ and $\Pi 2$ with Cartesian equation:

$$\Pi 1: x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$$

$$\Pi 2: x^2 + y^2 + z^2 + x + y + z - \frac{1}{4} = 0$$

(b) Calculate the distance between the centres of the two spheres above.

5. (6+3=9 marks)

Consider the system of equations:

$$-4x + y + 7z = -6$$

$$2x + y - 3z = 4$$

$$x + 2y + kz = m$$

- (a) Determine the conditions on k and m for which
- (i) the system has no solutions;
 - (ii) the system has only one solution;
 - (iii) the system has an infinite number of solutions.
- (b) In the case where the number of solutions is infinite, determine the general solution of the system of equations in Cartesian form.

6. (1+2+2+3+2+2+2+2=16 marks)

In this question, distance is in kilometres and time is in hours.

Two small drones, Drone A and Drone B are flying at steady speeds in straight lines.

At 14:00 Drone A is at the point (1, 1, 6). Its position vector \tilde{r}_1 , measured from an origin at ground level, after t hours is given by

$$\tilde{r}_1 = \langle 1, 1, 6 \rangle + t \langle 2, -2, 1 \rangle$$

(a) (i) Write down the velocity vector of Drone A.

(ii) Determine the speed of Drone A.

(iii) Assuming Drone A has been flying with this velocity since 12:00, what were Drone A's coordinates at 12:00?

At 14:00 Drone B is at the point (4, 7, 7). After four hours it is at the point (16, -29, 11).

(b) Show that the position of Drone B after t hours is given by

$$\tilde{r}_2 = \langle 4, 7, 7 \rangle + t \langle 3, -9, 1 \rangle$$

The paths of the two drones take them directly over the top of the same building (at different time).

Given that the top of this building is 500 metres above the ground.

- (c) (i) Determine the coordinates of the top of the building.
(Hint: the two drones are directly on top of this building when x-coordinate and y-coordinate of Drone A and B are the same.)

- (ii) Which drone passes over the top of the building first and what time does it do so?

(iii) Determine if the paths of the two drones intersect? Show your working.

(d) Calculate the angle between the paths of the two drones, giving the answer to the nearest degrees.

End of Section 2